Lasso and Model Order Selection. In-class Exercise 2

EL-GY 6143 Intro Machine Learning. Prof. Sundeep Rangan

Question

We wish to fit a linear model of the form,

$$\hat{y} = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p,$$

for a problem with $p = 100$ variables. Suggest regularizers $\phi(\boldsymbol{\beta})$ to impose the following constraints:

(a) All the coefficients $\beta_j, j = 1, \ldots, p$ should be close to zero, but not necessarily exactly zero.

(b) Most of the coefficients $\beta_j, j = 1, \ldots, p$ should be exactly zero

(c) Among the first fifty coefficients, $\beta_j, j = 1, \ldots, 50$, most coefficients should be zero. But, the other coefficients should be unconstrained.

Solution

(a) Use the L2-norm, $\phi(\beta) = \|\beta\|_2^2 = \sum_{j=1}^{p} |\beta_j|^2$

(b) Use the L1-norm, $\phi(\beta) = \|\beta\|_1 = \sum_{j=1}^{p} |\beta_j|$

(c) Only regularize the first 50 coefficients, $\phi(\beta) = \sum_{j=1}^{50} |\beta_j|$